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# Impacts économiques des risques industriels majeurs : Evaluation de la perte annuelle probable d'une entreprise marocaine

# Economic impacts of major industrial risks : Assessment of the probable annual loss of a Moroccan company

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## Résumé

Cet article évalue les impacts économiques des risques industriels majeurs sur l'activité économique d'une entreprise industrielle marocaine. Sur une base de données, répertoriée par un expert en risques industriels, indiquant le sinistre et sa gravité, il est constaté à l'aide d'une cartographie standard des risques que nous sommes confrontés à des risques très rares à fort impact. Cependant, cette analyse qualitative est insuffisante pour décrire le comportement de l'ensemble du portefeuille des grands risques industriels. En effet, la quantification de ces risques demeure nécessaire pour compléter l’analyse des risques. Pour y remédier, l'article analyse les différentes méthodes de construction et la répartition des valeurs de pertes des entreprises industrielles. L'estimation des pertes annuelles pouvant survenir est établie grâce à l'utilisation de deux principaux modèles statistiques, le premier : le modèle de risque individuel, le second : la simulation de Monte Carlo combinée à l'approche forfaitaire.


**Mots clés :** risque ; modélisation des risques ; modèle de risque individuel ; simulation Monte Carlo ; approche forfaitaire.


## Abstract

This article assesses the economic impacts of major industrial risks on the economic activity of a Moroccan industrial company. On a database, listed by an expert in industrial risks, indicating the disaster and its severity, it is established using standard risk mapping that we are faced with very rare risks with great impact. However, this qualitative analysis is insufficient to describe the behavior of the entire portfolio of major industrial risks. Indeed, the quantification of these risks remains necessary to complete the risk analysis. To remedy this, the article analyzes the different construction methods and the distribution of industrial enterprise loss values. The estimate of the annual losses that may occur is established through the use of two main statistical models, the first: the individual risk model, the second: the Monte Carlo simulation combined with the flat-rate approach.


**Keywords :** risk ; risk modeling ; individual risk model ; Monte Carlo simulation ; flat-rate approach.

## Introduction

Risk management is widely adopted and deployed by companies through the development of theories and practical tools. It is present everywhere at levels of intensity that vary according to the size and strategic objectives of the organization. The interest in controlling and reducing the impact of risks testifies to the civil and ecological commitment of the economic actor. However, it is very difficult, despite the security and prevention measures, to avoid the materialization of the disaster. The impact of major industrial risks on the profitability of industrial companies is among the great debates in risk management theory. Generally, industrial companies are less interested in preventing risks for which they have taken out insurance coverage (Marsden, 2014). At the same time, insurance companies have an interest in adopting co-insurance so that they can reduce the risk premiums to be covered for these industrial companies (Bellouq, 2022).

We are interested in this article in the modeling of major risks, and this in the absence of exhaustive data describing the scenarios of these risks. Major risk is one of the major categories of risk. This type of risk is characterized by a very low probability of occurrence and by a significant financial impact. An example of a major risk for a Moroccan industrial company: "*Loss of the main stone removal room containing the control cabinets and the power outlets in a fire. Total shutdown of the Benguerir mine for 10 days then restart in degraded mode during the end of the reconstruction phase of 1 month and 20 days. Operating loss equivalent to one month of total shutdown of Benguerir mining. Cost of rebuilding the hall, 4.000.000,00 MAD*". This disaster occurs once every 62.5 years and generates a loss of 207.750.000,00 MAD. Such risk may not be the only risk of this business. We can find a hundred major risks within a company. If we imagine this situation, we will find the need to quantify these risks in order to be ready to cover them if it falls within the solvency limits of the company. However, the decision to insure is based on several considerations: identification of the risks to be covered, evaluation of the total annual losses, measurement of the risks, etc.

In the dynamics of monitoring its industrial activities, a Moroccan company with industrial activities calls on renowned experts in industrial risk management in order to identify and assess probable risks as well as the development of a protection and prevention roadmap. Indeed, this collaboration gave birth to a risk matrix which lists all claims by associating an average impact and probability of occurrence calculated on the basis of data on international companies whose industrial activity is similar to that of the company studied. We name the n-row, 3-column matrix n-risk portfolio. The first column is the risk number, the second corresponds to its average probability of occurrence and the third to its associated average impact.

Certainly, this matrix alone makes it possible to carry out a qualitative analysis but once you feel the need to quantify and estimate parameters, these data are transformed into references. The problem relating to the lack of data leads to the following main question: Can we, from reduced data, model the total annual loss of a portfolio?

The absence of an exhaustive history of risks points to the need to foresee methods adapted to this kind of data. This is where statistical tools resonate to produce exploitable knowledge in this direction. The methodology adopted in this article essentially consists of analyzing the data provided by the expert and deploying a whole procedure for modeling these risks by examining all the possible methods and explaining the limits of the approaches which did not allow us to lead to satisfactory results.

To do this, this article will be structured around the following two sections: the first section will draw up a selective empirical literature review dealing with the different methods of construction of the distribution of the probable total losses generated by industrial risks, the second section will present the interpreted data and results of the individual risk model and the Monte Carlo simulation combined with the flat-rate approach.

## 1. Selective empirical literature review

The empirical literature relating to the assessment of the probable total loss of a set of major risk portfolios has received less attention compared to recurrent risks. Moreover, much of this empirical literature has focused on developed countries (USA, Europe, Japan) much more than developing countries. The lack of an exhaustive history of risks is one of the main causes of this dearth of studies in these countries.

In this context, this section draws up a review of the empirical literature dealing with the probabilistic models describing the distribution of the random variable which represents the amount of the total annual loss of a set of major risk portfolio.

There are mainly two risk models to model the total amount of losses per portfolio namely: the individual risk model and the collective risk model. Indeed, the choice between the two models comes down to the type of random variables presenting the contracts.

### 1.1. Individuel risk model

Dickson, DCM (2005) defines the individual risk model as follows, we consider a portfolio composed of a fixed number, n, of independent contracts, and the total amount claimed on the portfolio in a fixed period of time is modeled as a random variable T, given by:

$$T = Y_1 + Y_2 + Y_3 + \cdots + Y_n$$

Where is the total amount claimed on the contract I and $Y_1, \ldots, Y_n$, are assumed to be independent, but not necessarily identically distributed.

However, it turns out that handling this model is more difficult than expected even if at first glance this quantity seems simple in terms of numerical calculations and in terms of obtaining analytical expressions for the distribution of T.

The second model is the collective risk model (or the global risk model), where Dickson, DCM (2005) models the successive claims arising from the portfolio of random variables$Y_1$ $Y_2$, …,$Y_N$ ., independent, identically distributed by ignoring which contract gave rise to what claim. The number of requests during the fixed period of time is a random variable N which is assumed to be independent of $Y_i$.

The total amount of the demand (or global claims) is modeled as a random variable T given by:

$$T = Y_1 + \cdots + Y_N$$

Csörg″o, S. and Teugels, JL (1990) consider that the distribution of T is only one example of a compound distribution. This property makes the manipulation of this model much easier knowing the distribution of N and $X_i$ for the collective risk model.

In the general model, they are modeled as a function of the occurrence of the claim and a random variable Zi which corresponds to the amount claimed if the claim i occurs. $Y_i$

For i = 1,…, n:

$$Y_i = \begin{cases} 0 \; with \; probability \; 1 - q_i \\ Z_i with \; probability \quad q_i \end{cases}$$

❖ **Determination of the expectation and variance of T**

We will start by stating the general formula for the expectation and variance of the individual risk model. Let $E(Z_i) = \mu_i$ and $Var(Z_i) = \sigma^2{}_i$ assumed to be finite

We can at this point determine those of Yi and T:

$$E(Y_i) = (1 - q_i) \times 0 + q_i E(Z_i) = q_i \mu_i$$

Then,

$$\mathrm{E(T)} = \sum_{i=1}^{n} \mathrm{E(Y_i)} = \sum_{i=1}^{n} \mathrm{q_i \mu_i}$$

Now let's move on to the variance:

$$\begin{aligned} \mathrm{Var\ (Y_i)} &= \mathrm{E(Y_i{}^2)} - (E(\mathrm{Y_i}))^2 \\ &= (1 - \mathrm{q_i}) \times 0^2 + \mathrm{q_i E}\left(\mathrm{Z_i{}^2}\right) - \mathrm{q^2{}_i \mu^2{}_i} \\ &= \mathrm{q_i}\left(\sigma_i{}^2 - \mu^2{}_\mathrm{i}\right) - \mathrm{q^2{}_i \mu^2{}_i} \\ &= \mathrm{q_i}\sigma_i{}^2 + \mathrm{q_i}(1 - \mathrm{q_i})\mu^2{}_\mathrm{i} \end{aligned}$$

Given the independence of the risks, the variance of T is:

$$Var(T) = \sum_{i=1}^{n} Var(Y_i) = \sum_{i=1}^{n} (\,q_i\sigma_i{}^2 + \,q_i(1 - \,q_i)\mu^2{}_i)$$

Assuming that the risks in a portfolio are independent random variables, the distribution of their sum can be calculated by making use of the convolution product. It turns out that this technique is quite laborious especially in the absence of an explicit form of the density function of, then there is a need for other methods. One of the alternative methods is to use functions of moments as illustrate Grimmett and Stirzaker (2001) to generate transformations related to the characteristic functions and functions generating probabilities.

Sometimes it is possible to recognize the moment generating functions of a random sum independent of the variables and hence identify the distribution function. And in some cases, one can successfully employ a technique called the Fourier transform introduced by Grübel, R (1989) and then developed by Pitts, S. M (2006), fast to reconstruct density from a transform.

There is also a totally different approach, developed by DeGroot and Schervish (2002), is to use approximations of the distribution of the total amount of annual loss S.

The determination of the distribution of the sum of independent random variables can often be facilitated by the use of the transformations of the distribution function. It is in this sense that the moment generator function for a positive random variable comes.

$$m_X(t) \;=\; E(e^{tX}) \quad -\infty < t \;<\; h \qquad (\,h \;>\; 0\,)$$

For X and Y two independent random variables, the moment generating function of X + Y is expressed by:

$$m_{X+Y}(t) = E\left(e^{t(X+Y)}\right) = E(e^{tX}) \times E(e^{tY}) = \; m_X(t) \;\times\; m_Y(t)$$

It is often difficult to identify this moment with that of a usual compound distribution. Indeed, this is due to the variation of$q_i$ and $b_i$ When these two parameters are constant, it is possible to identify the moment of a compound binomial distribution.

This is why DeGroot and Schervish (2002) lean towards approximations in order to solve this problem.

There are two methods of approaching the individual risk model, namely:

- The approximation by the normal distribution
- The transformation of the individual risk model into a collective risk model and the application of the algorithm of Panjer, HH (1981).

❖ **Approximation of the individual risk model by the normal distribution:**

It is obvious that the individual risk model is conceptually simple because it is nothing but the sum of n independent random variables, and as such is easy to deal with in principle.

However, as soon as the size n increases, the stones become heavier and heavier. This to encourage specialists in the field to consider the use of approximations to better understand the distribution of T. We are going to implement more than a single approximation starting with the simplest which is none other than the normal distribution. .

DeGroot and Schervish (2002) believe that the normal approximation is justified by the central limit theorem for large n. Here, the distribution of T is approximated by a normal distribution of mean and variance $E\ (T)$ and $Var\ (T)$ respectively, given:

$$E\ (T) = \sum_{i=1}^{n} q_i \mu_i$$

$$Var\ (T)\ = \sum_{i=1}^{n} (\ q_i \sigma_i{}^2 +\ q_i(1 -\ q_i)\mu^2{}_i\ )$$

We obtain the following approximation:

$$T \sim\ N(\textstyle\sum_{i=1}^{n} q_i \mu_i\, , \sum_{i=1}^{n} (\ q_i \sigma_i{}^2 +\ q_i(1 -\ q_i)\mu^2{}_i))$$

### 1.2. Approximation of the individual model by the collective model

The manipulation of the collective model is much simpler than that of the individual model thanks to the type of random variables of the collective model estimated by Dickson, DCM (2005) which are independent and identically distributed.

Indeed, this approach is called the approximation by the compound Poisson distribution CP(λ,F), adopted by Embrechts, Jensen, Maejima and Teugels (1985).

To apply this approach, it is first necessary to clarify the expression of λ and F according to the parameters of the individual risk model.

The approach consists in replacing the Bernoulli distribution($q_i$) by poisson($\lambda_i$) where $\lambda_i$=$q_i$

Thus, the new distribution of $Y_i$ will be $CP(q_i, , F_{Z_i})$ for i = 1, .., n.

Let us state the theorem of Embrechts, Jensen, Maejima and Teugels (1985) which justifies the existence of a distribution for the sum of the random variables which follow the Compound Poisson distribution:

Let, $S_1$ ,…, $S_n$be independent random variables which follow Compound Poisson distribution of respective parameters,…, and respective $\lambda_1$ ,…, $\lambda_n$ elementary distribution functions, $F_1, \dots, F_n$. Then T = $S_1$ +…+ $S_n$ follows a compound Poisson distribution with parameter Poisson $\lambda$ and F.

With $\lambda = \lambda_1 + \cdots + \lambda_n$ and $F(x) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} F_i(x)$

Under these assumptions we can apply one of the best known recurrence methods in insurance, namely the recurrence of Panjer, HH (1981). Indeed, this formula is applicable and valid only for collective risk models, we will present the Panjer method below:

Panjer's algorithm is a recursive relation which allows to calculate the probability mass function of S for a family (a, b, 0) of frequency distributions

$$S = \begin{cases} \sum_{j=1}^{N} Y_j \; si \; N > 0 \\ 0 \;\; si \; N = 0 \end{cases}$$

The recursive relation is given by:

$$f_S(x) = \frac{\sum_{y=1}^{x} (a + b\frac{y}{x}) f_Y(y) f_S(x-y)}{1 - a f_Y(0)}$$

With: $f_S(0) = P(S = 0) = M_N\{log(f_Y(0))\}$

Only the Poisson, Binomial and Negative Binomial distributions are members of this family (a,b, 0).

Poisson distribution P($\lambda$) : a = 0, and b = $\lambda$

Binomial distribution B(n, q) : $a = -\frac{q}{1-q}$ et b $= (\mathrm{n} + 1)\frac{q}{1-q}$

Negative Binomial Distribution BN(r,$\beta$ ) : a = $\frac{\beta}{1+\beta}$ et b =(r-1)$\frac{\beta}{1+\beta}$

There is also in the literature a practical technique which makes it possible to construct the distribution of loss values, that of the monte carlo simulation combined with the fixed approach, developed by Plato, V. and Constantinescu, A (2014).

The prince of the flat-rate approach is quite simple: The random variable X represents the claims costs for an individual risk during a fixed period (eg: one year). The definition of X is given by:

Impact = X = $\begin{cases} \text{B if I} = 1 \\ \text{0 if I} = 0 \end{cases}$

With:

I =$\begin{cases} \text{1 if at least one accident occurs} \\ \text{0 else} \end{cases}$

The random variable corresponds to the total cost of claims incurred for the contract if at least one claim occurs during the period. In health care insurance, the random variable B is the sum of all costs incurred if at least one claim is made during a period.

Monte Carlo simulation is a simulation technique that combines data on the frequency of occurrence and impact of individual risks.

- **Presentation of the generation approach:**

Let $S = f(X_1, X_2, \dots, X_k)$ with $X_j$ random variables. We consider a risk $R_j(p_{donneé\ j}, Imp_j)$ with $p_{donneé\ j}, Imp_j$ the probability of occurrence and the impact estimated by the expert.

The following steps are followed to generate annual losses:

Generate n samples of probability of occurrence of each risk, $j$, $p_{j1}, p_{j2}, \dots p_{jn}$ ~ Bernoulli (n,$p_{donneé\ j}$ ).

Proceed to the allocation of charges resulting from the occurrence of the claim in question.

## 2. Case study: annual global impact of major risks on the economic activity of a moroccan industrial company

Based on the work drawn up in the previous section, this work aims to assess the economic impacts of major industrial risks on the economic activity of a Moroccan industrial company.

The choice of case study is based on several considerations. First, the Moroccan industrial sector has embarked on a dynamic of economic growth which has been greatly consolidated since the implementation of the Emergence Plan and the conclusion, in 2009, of the National Pact for Industrial Emergence. To date, undeniable achievements should be noted: the 22%

increase in exports from the industry sector, a marked improvement in infrastructure and the establishment of world industrial leaders, increasing foreign direct investment (FDI) to at an average annual rate of 23% since 2009. Moreover, these performances have made it possible to better position Morocco on the radars of the planet as a reliable and competitive industrial destination. It is now a question of confirming the foundations of the industrial building in place, in order to make optimal use of the industrial potential of the country which is placed positively at the crossroads of America, Europe, Africa , and the Middle East.

This is why it is useful to choose a large Moroccan industrial company as a case study in order to assess the vulnerable points of the sites belonging to it, because its massive industrial capacity coupled with the flexibility of its productive apparatus does not provide immunity from risk.

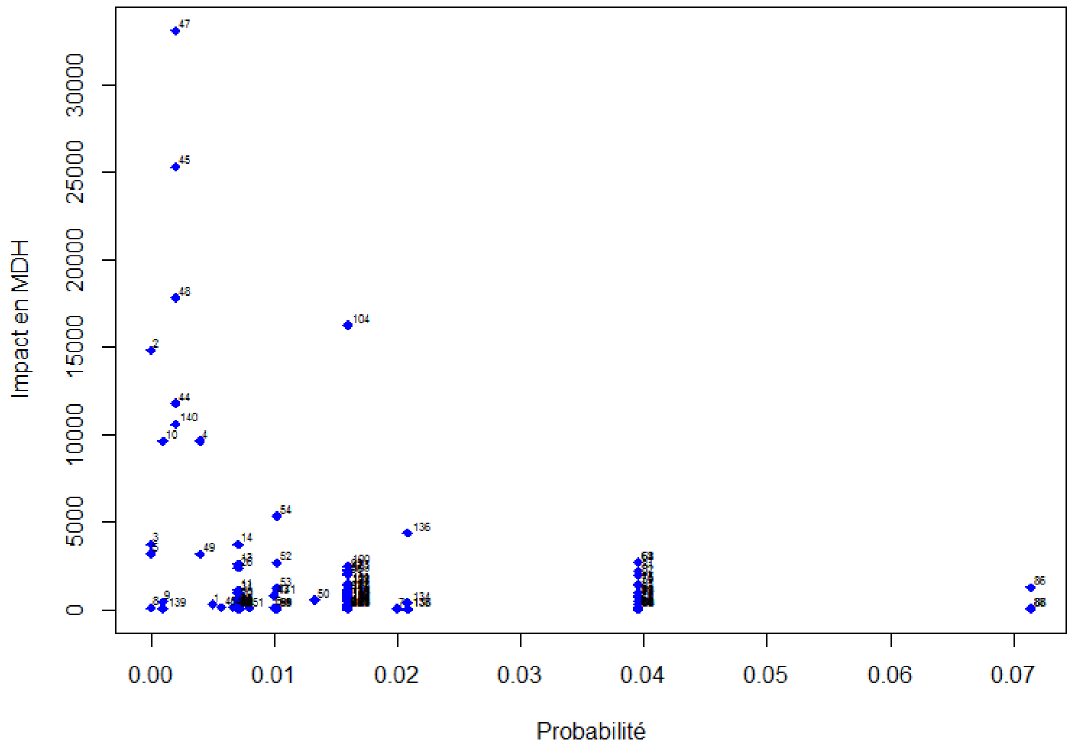
### 2.1. Study data

Each risk has an average impact and an average probability of occurrence. We therefore have a matrix of n = 141 rows and 3 columns. Risks are represented in rows, while the first column refers to the risk number, the second column to its estimated probability of occurrence and the third column to its estimated average impact.

### 2.2. Qualitative approach to industrial risk management

The risk analysis process comprises two phases. These are the qualitative and quantitative approaches to risk assessment. We will start intuitively with the qualitative risk analysis. This approach is purely descriptive. It consists of positioning risks according to their respective probability and impact in order to then be able to define the type of risk according to the standard risk mapping:

**Figure N°1 : Mapping of the major risks of the industrial company**




**Source : Established using R software**

As you can see from Figure N°1 the overwhelming majority of risks are characterized by a probability of occurrence of less than 4% and an impact of more than 1000 Million MAD. Given this observation, we are certain that these are major risks. However, the quantification of these risks remains necessary to complete the risk analysis. So we will discuss in detail below everything related to modeling. We will try to model the losses generated by the realization of the risks constituting the portfolio according to several approaches.

### 2.3. Definition of variables

The empirical analysis is conducted using a 141-variable model. The estimate of the probable annual loss relates to a portfolio of 141 risks listed by the expert.

141 variables are used in the estimation.

We have chosen the variable $\mathrm{Y_i}$ to model the loss due to risk i which is defined as follows:

$$\mathrm{Y_i} = \begin{cases} 0 \text{ with probability } 1 - q_i \\ b_i \text{with probability } \quad q_i \end{cases}$$

with: the constant $\mathrm{b_i}$ which represents its impact and $\mathrm{q_i}$ its probability of occurrence.

### 2.4. Choise of model

Given the type of database we have, we can only work with a short-term model according to Dickson, DCM (2005). Thus, we have to choose between the individual risk model and the collective risk model. The choice in our case is quite simple since the random variables with which we are going to operate are independent but not identically distributed, hence the need to choose the individual risk model.

### 2.5. Individuel risk model application

In our model there is a fixed number n = 141 of risks, and, for i = 1,…, n the amount of loss in the event of the occurrence of risk i is represented by the random variable Yi. We assume that the Yi are positive independent random variables, but they are not identically distributed. The total amount claimed across the entire portfolio within the fixed time period (one year) is:

$$T = Y_1 + Y_2 + Y_3 + \cdots + Y_n$$

For risk i, the probability of registering exactly one occurrence is $q_i$ (independent of other risks).

If a claim i has occurred then the amount of loss is known and fixed $b_i$ (> 0).

In this case, since Yi is a discrete random variable formulated as follows:

$$Y_i = \begin{cases} 0 \; with\; probability\; 1 - q_i \\ b_i with\; probability\; q_i \end{cases}$$

So $T = Y_1 + \cdots + Y_n$ is also a discrete random variable.

- **Determination of the expectation and variance of our model**

In our case we have $Z_i = b_i$ deterministic, hence according to the properties of the expectation and the variance we have:

$E(Z_i) = b_i$ and $Var(Z_i) = 0$

So we will have for $Y_i$ and T the following expressions:

$E(Y_i) = (1 - q_i) \times 0 + q_i E(Z_i) = q_i b_i$ and $Var(Y_i) = q_i(1 - q_i){b_i}^2$

$E(T) = \sum_{i=1}^{n} q_i b_i$ and $Var(T) = \sum_{i=1}^{n} Var(Y_i) = \sum_{i=1}^{n} q_i(1 - q_i){b_i}^2$

### 2.6. Determination of the model losses distribution

Since the risks in the portfolio studied are independent random variables, we can use moment functions like illustrate Grimmett and Stirzaker (2001) to generate transformations related to the probability generating functions. It is in this sense that the moment generator function for a positive random variable comes. After application of Transforms on our model we find:

$$m_{Yi}(t) = E(e^{tYi}) = 1 - q_i + q_i E(e^{tZi}) = 1 - q_i + q_i E(e^{tb_i})$$

$$m_T(t) = E(e^{tT}) = 1 - q_i + q_i E(e^{tT}) = \prod_{i=1}^{n} (1 - q_i + q_i E(e^{tb_i}))$$

We have not been able to identify this moment with that of a usual compound distribution. Indeed, this is due to the variation of $q_i$ and $b_i$, if these two parameters were constant we would have identified the moment of a compound binomial distribution. So to be able to apply the exact methods, it is necessary to know the explicit expression of the function of density of the variables $Y_i$. In our case, since we are dealing with binary random variables, which therefore have no density function, which makes these methods unusable.

This is why we will focus on the approximations established by DeGroot and Schervish (2002) in order to solve this problem.

There are two methods to approximate the distribution of T, either by the normal distribution approximation or the Compound Poisson distribution.

- **Approximation of the individual risk model by the normal distribution:**

It is obvious that the individual risk model is conceptually simple because it is nothing but the sum of n independent random variables, and as such is easy to deal with in principle.

However, as soon as the size n increases, the stones become heavier and heavier. This to encourage specialists in the field to consider the use of approximations to better understand the distribution of T. We are going to implement more than a single approximation starting with the simplest which is none other than the normal distribution. .

The normal approximation is justified by the central limit theorem for large n. Here, the distribution of T is approximated by a normal distribution of mean and variance $E(T)$ and $Var(T)$ respectively, given:

$$\mathrm{E}\ (\mathrm{T}) = \sum_{i=1}^{141} \mathrm{q_i} \mu_\mathrm{i}$$

$$\mathrm{Var}\ (\mathrm{T}) = \sum_{i=1}^{141} \mathrm{q_i}(1 - \mathrm{q_i})\mathrm{b^2}_\mathrm{i}$$

For our case we therefore have:

$$\mathrm{T} \sim N(\sum_{i=1}^{141} \mathrm{q_i b_i}, \sum_{i=1}^{141} \mathrm{q_i}(1 - \mathrm{q_i}) b_i{}^2)$$

Advantages of the normal distribution:

- Easy and quick implementation for the practical part;
- We only need $\mu_i$ *and* $\sigma_i$.

The code for the implementation of this method under R is as follows:

**Figure N°2 : R code of the normal approximation**

```
###################################TCL#######################################
#E correspond à la E(T)
E=sum(base[1:141,3]*base[1:141,2])

#var fait référence à var(T)
var=sum((base[1:141,3]^2)*base[1:141,2]*(1-base[1:141,2]))

#l'argument de  la fonction de de répartition de la loi normale ( 0,1 )
Phi <- function(x)((x-E)/sqrt(var))

#le graphe de la probabilité cummulé de la distribution de perte
curve(pnorm(Phi(x), mean=0, sd=1), from = 0, to = 30000)

#le graphe de la densité de probabilté de T selon l'approximation normale

plot(dnorm(seq(0,30000),E,sqrt(var)),col="blue",lwd="1")

#la value at risk d'ordre 99,5%
VaRTCL=qnorm(0.995)*sqrt(var) + E
```

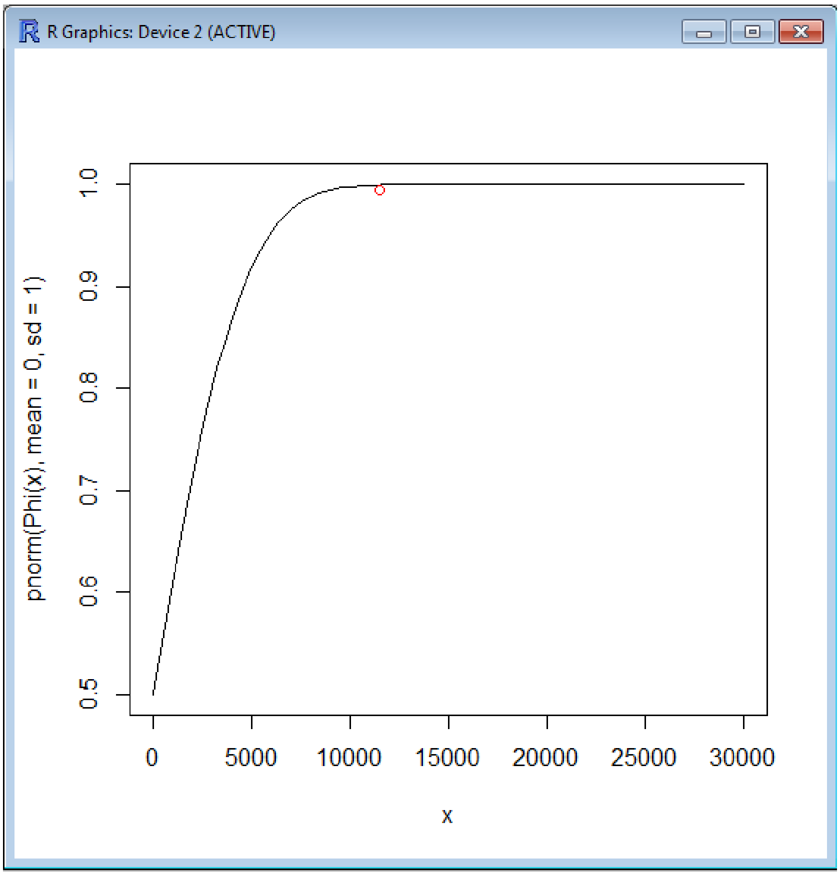
**Source : Established using R software**

As we can see from Figure N°2, this method does not require a laborious manipulation, only the knowledge of the expectation and the variance of the distribution of T allows to carry out the implementation:

```
> E
[1] 2315.656
> var
[1] 12567084
> sqrt(var)
[1] 3545.008
```

The parameters of the normal distribution are E and var calculated from the data in the database. We see that the average loss estimated by the expert is of the order of 2315.656 Million MAD.

**Figure N°3 : Graphical representation of the distribution function of T**



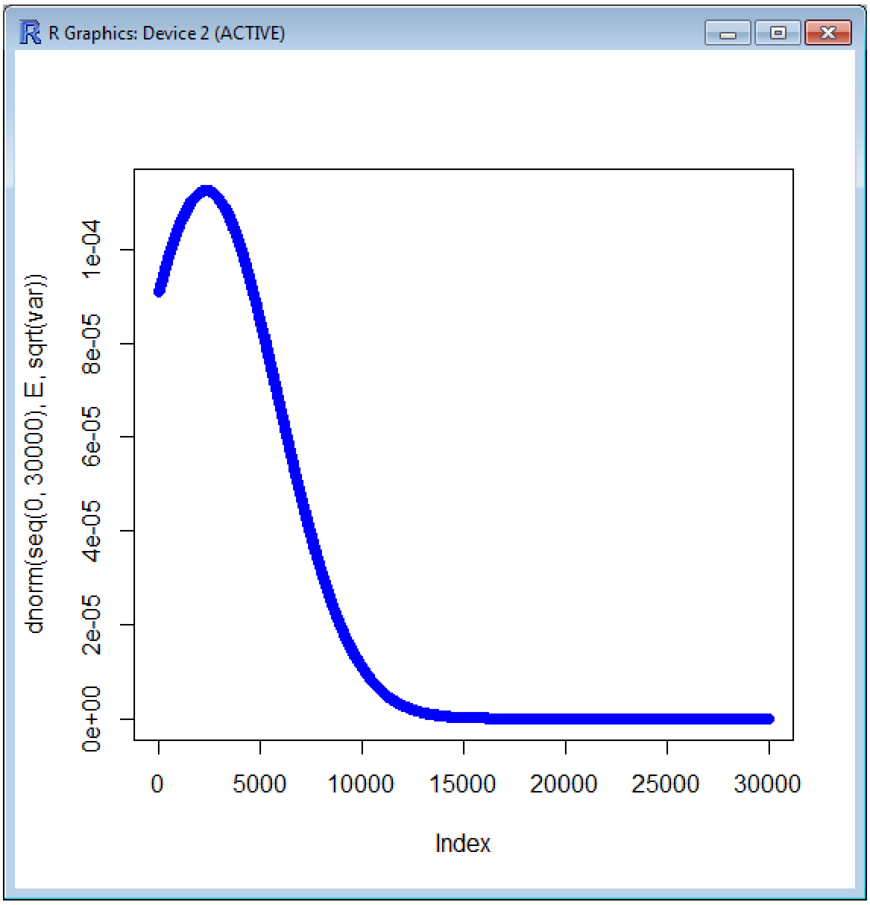

**Source : Established using R software**

Figure N°3 shows the variation of the distribution function of the total annual loss amount. This is the cumulative probability graph, for example the probability that the amount of loss is less than 5000 Million MAD is 0.8. You can notice a red point on the graph, it corresponds to the value at risk of order 99.5%. Indeed this point corresponds to the value 11446.99.

```
> VaRTCL
[1] 11446.99
```

Let's move on to the visualization of the probability density graph of the probable loss amounts. The following Figure N°4 represents the distribution of the variable T :

**Figure N°4 : Probability density of the approximation of T**




**Source : Established using R software**

The non-negativity of the loss amounts made it necessary to take into account only one part of the density graph. Indeed we are only interested in positive quantiles because the amount of losses cannot be negative.

❖ **Approximation of the individual model by the collective model**

When we found in the literature a method which consists in transforming the individual risk model into a collective risk model, we did not hesitate to apply it to enrich our work. Indeed, this approach is called the approximation by the compound Poisson distribution CP (λ, F).

To apply this approach we must first clarify the expression of λ and F as a function of the parameters of the individual risk model.

The approach consists in replacing the Bernoulli $(q_i)$ distribution by Poisson $(\lambda_i)$ where $\lambda_i = q_i$.

Thus, the new distribution of $Y_i$ will be $CP(q_i, F_{Z_i})$ for i = 1, .., n.

Let us state the theorem which justifies the existence of a distribution for the sum of the random variables which follow the Compound Poisson distribution.

At this stage we can move on to determining the distribution of T which is none other than $F_T \approx$CP ( $q_+, F$).

With $q_+ = q_1 + \ldots + q_n$ and $F(x) = \sum_{i=1}^{n} \frac{q_i}{q_+} F_{Z_i}(x)$ .

- **Practical side :**

The impacts of the risks on which we are working are not different, which violates one of the assumptions of this approximation. Indeed, in order to be able to generate realizations of T from Compound Poisson distribution, it is necessary to apply the algorithm of Panjer, H. H (1981). In the case of discrete random variables, there is a basic condition to respect, which is that $b_i$ the must be all distinct in order to be able to use this algorithm. In our case, this hypothesis is not verified so we cannot go further on this track.

We still implemented this method to see how important this assumption is.

The R code of the implementation of the Panjer algorithm applied to compound Poisson distribution is described is Figure N°5 :

**Figure N°5 : Panjer's R code applied to the collective model**

```
######################## Panjer appliqué au modèle collectif########################

#le montant de perte maximal
maxS=sum(base[1:141,3])

#le lambda du modèle CP
qplus=sum(base[1:141,2])

#Création et initialisation de la matrice des probabiltés de S par 0
S<-rep(0,maxS)

#P(S=0)=exp(-lambda)
S0=exp(-qplus)

#Implémentation de Panjer

for (r in 1:maxS){
for (j in 1:min(r,141)){
if (r-j>0) {S[r]=S[r]+j*base[j,2]*S[r-j]} else {S[r]=S[r]+j*base[j,2]*S0}
}
S[r]=qplus*S[r]/r}
```

**Source : Established using R software**

**Figure N°6 : Extract from Panjer's results applied to the collective model**

Data: S

| | x |
|---|---|
| 1 | 9.598386e-04 |
| 2 | 1.266100e-05 |
| 3 | 7.789439e-06 |
| 4 | 7.679720e-04 |
| 5 | 1.194034e-05 |
| 6 | 1.919835e-03 |
| 7 | 3.864484e-03 |
| 8 | 5.482001e-05 |
| 9 | 1.929092e-04 |
| 10 | 2.149215e-04 |
| 11 | 1.406340e-03 |
| 12 | 1.406944e-03 |
| 13 | 1.483987e-03 |
| 14 | 1.485235e-03 |
| 15 | 1.402410e-03 |
| 16 | 1.411420e-03 |
| 17 | 1.442903e-03 |
| 18 | 1.504917e-03 |
| 19 | 1.506144e-03 |
| 20 | 1.510881e-03 |
| 21 | 1.513775e-03 |
| 22 | 1.525610e-03 |
| 23 | 1.539070e-03 |
| 24 | 1.553533e-03 |
| 25 | 1.568011e-03 |

Data: S

| | x |
|---|---|
| 9768 | 1.353400e-84 |
| 9769 | 1.315546e-84 |
| 9770 | 1.278751e-84 |
| 9771 | 1.242983e-84 |
| 9772 | 1.208215e-84 |
| 9773 | 1.174418e-84 |
| 9774 | 1.141565e-84 |
| 9775 | 1.109631e-84 |
| 9776 | 1.078588e-84 |
| 9777 | 1.048414e-84 |
| 9778 | 1.019082e-84 |
| 9779 | 9.905703e-85 |
| 9780 | 9.628553e-85 |
| 9781 | 9.359149e-85 |
| 9782 | 9.097274e-85 |
| 9783 | 8.842718e-85 |
| 9784 | 8.595278e-85 |
| 9785 | 8.354754e-85 |
| 9786 | 8.120953e-85 |
| 9787 | 7.893687e-85 |
| 9788 | 7.672775e-85 |
| 9789 | 7.458038e-85 |
| 9790 | 7.249304e-85 |
| 9791 | 7.046406e-85 |
| 9792 | 6.849181e-85 |

**Source : Established using R software**

The fact of working with discrete random variables makes the handling of the simplest actuarial approaches very complicated. As you can see from Figure N°6, the probabilities of amounts in the order of a few thousand Million MAD are very low. So we can only be satisfied with the results provided by the normal approximation of the individual risk model and see another avenue. We have thought of a simulation approach that is not based on specific assumptions.

### 2.7. Monte Carlo simulation and lump sum approach

Before talking about modeling, we must consider a method to create a database of loss occurrences that will form the starting point for everything that follows. To do this, we will use the Monte Carlo simulation combined with the flat-rate approach developed byPlato, V. and Constantinescu, A (2014).

**Flat-rate approach:**

We consider the random variable X is given by:

$$\text{Impact} = X = \begin{cases} B \text{ if } I = 1 \\ 0 \text{ if } I = 0 \end{cases}$$

With:

$$I = \begin{cases} 1 \; if \; at \; least \; one \; accident \; occurs \\ 0 \; else \end{cases}$$

In our case we are going to consider I that we will note $p_{ij}$ the probability of occurrence of risk i under the conditions of scenario j and $Imp_i$ that we will note as being the impact of risk i estimated by the expert.

In our case, the variable X has the following characteristics:

E(X) = q E(B), Var(X) = E (I)Var(B) + Var(I)$E^2$(B)

$M_X(t) = qM_B\,(t) + 1 - q$

$Imp_i$ which will be noted as the impact of risk i estimated by the expert.

**The Monte Carlo method:**

It was thought to exploit this technique in order to approximate the distribution of the global annual losses. Thanks to its statistical strength, it will allow us, by reasoning on the probability scenarios of the occurrence of risks, to define the overall annual loss generated by the realization of scenario i. We are going to simulate 100,000 random draws per scenario.

Knowing that we operate with fairly low estimated probabilities of occurrences (varying from once every 14 years to once every 100,000 years), it seemed useful to us to reason about the occurrence or not of the disaster per scenario, which equivalent to considering that the annual frequency of occurrence is either 0 or 1 which will generate from a Bernoulli distribution of parameter the probability of occurrence estimated by the expert for each risk respectively.

In principle, this involves simulating 141 drawdowns of the realizations of each risk over a calendar year. This procedure will be reproduced a very large number of times for our case we will simulate a total of 1 million years in order to be able to go through all the possible scenarios.

$p_{ij}$ : the probability simulated in scenario i for risk j

$Imp_{ij}$ : the impact of simulated risk j if we place ourselves under the conditions of scenario i

If the risk is judged by the expert as a single loss then the allocation of the load is made as follows:

$$Imp_{ij} = \begin{cases} Imp_j, if \;\; p_{ij} = 1 \\ 0 \;\; , else \end{cases}$$

However, for the case of multiple risks grouped together in a category, we must perform an equiprobable drawdown of an amount among those of the same category. It is .$\mathrm{Imp}_{\text{tiré}}$

$$Imp_{i,classek} = \begin{cases} Imp_{tiré}, if \;\; p_{ij} = 1 \\ 0 \;\; , else \end{cases}$$

As you can see the allocation of the load is done by category.

Scenario i of estimated annual losses:

$$S_i = \sum_{j=1}^{n} Imp_{ij}$$

By going through all the scenarios, we will obtain an approximate value of the overall loss for each scenario. So we can say that we have built the working basis which is nothing other than the distribution of losses.

Practical side :

We started with the construction of the matrix of grim achievements first. The principle is quite simple we generated the scenarios for the realization of the risk classes, we the expert to identify 60 risk classes.

Then we will go to the allocation of the load of the class, for the classes formed by a single risk we will take the value of the impact associated with the risk. Whereas for the other type of class we will carry out an equiprobable pulling of the impact from the set of impacts of the corresponding class. The allocation of the load will be done scenario by scenario.

The R code displayed in Figure N°7 is that of the simulation of the annual scenarios of loss amounts implemented using the approach explained in the previous part.

**Figure N°7 : Monte-Carlo simulation under R**

```
##########################Simulation de Monte Carlo##########################

#Nombre de Scénarios
n=(10^5)+1

#Nombre de classes de fréquence
NCLASSE=21

#La matrice des scénarios
I= matrix(nrow=NCLASSE,ncol=n)

#La génération des réalisations des risques pour l'ensemble des scénarios
j=1
i=1
while(i<142)
{ if(base[i,4]==1)
   {  I[j,]=rbinom(n,1,base[i,2])
      I[j,1]=i
      j=j+1
      i=i+1 }
  else
  {
    I[j,]=rbinom(n,1,base[i,2])
    I[j,1]=i
    i=i+base[i,4]
    j=j+1
  }
}


#L'attribution de la charge individuelle des risques

for(j  in 1:NCLASSE)

{if((base[I[j,1],4]==1))

{ I[j,2:n]=base[I[j,1],3]* I[j,2:n] }
if(base[I[j,1],4]!=1)
 { for(k in 2:n)
    {I[j,k]=base[I[j,1]+round(runif(1, min=0, max=base[I[j,1],4]-1)),3]* I[j,k]}
 }
}

#le vecteur des montants de pertes annuels par scénario
ss= matrix(1,nrow=1,ncol=NCLASSE)%*%I[]
```

**Source : Established using R software**

**Figure N°8 : Extract from the risk matrix**

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 18 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 12 | 49 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 13 | 50 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 14 | 53 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 15 | 56 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 16 | 60 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 17 | 61 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 16255.43 | 0 | 0 |
| 18 | 103 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 19 | 104 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 372.68 | 0.00 | 0 | 0 |
| 20 | 109 | 0 | 0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |
| 21 | 138 | 0 | 0 | 46.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 | 0.00 | 0 | 0 |

**Source : Established using R software**

Matrix I represents the amounts of losses generated by the materialization of the claim by scenario. In fact, from Figure N°8, for single claims, the realization scenarios have either 0 or the impact of the theoretical risk (estimated by the expert). However, for the claims that we have grouped by identical frequency we will find either 0 or an amount drawn by a discrete uniform distribution equiprobable between the impacts of the same group.

**Figure N°9 : Extract from the vector of annual losses**

| | V1 |
|---|---|
| 1 | 885.00000 |
| 2 | 103.20000 |
| 3 | 0.00000 |
| 4 | 0.00000 |
| 5 | 0.00000 |
| 6 | 0.00000 |
| 7 | 0.00000 |
| 8 | 0.00000 |
| 9 | 0.00000 |
| 10 | 0.00000 |
| 11 | 0.00000 |
| 12 | 50.50000 |
| 13 | 0.00000 |
| 14 | 0.00000 |
| 15 | 0.00000 |
| 16 | 0.00000 |
| 17 | 1921.90000 |

**Source : Established using R software**

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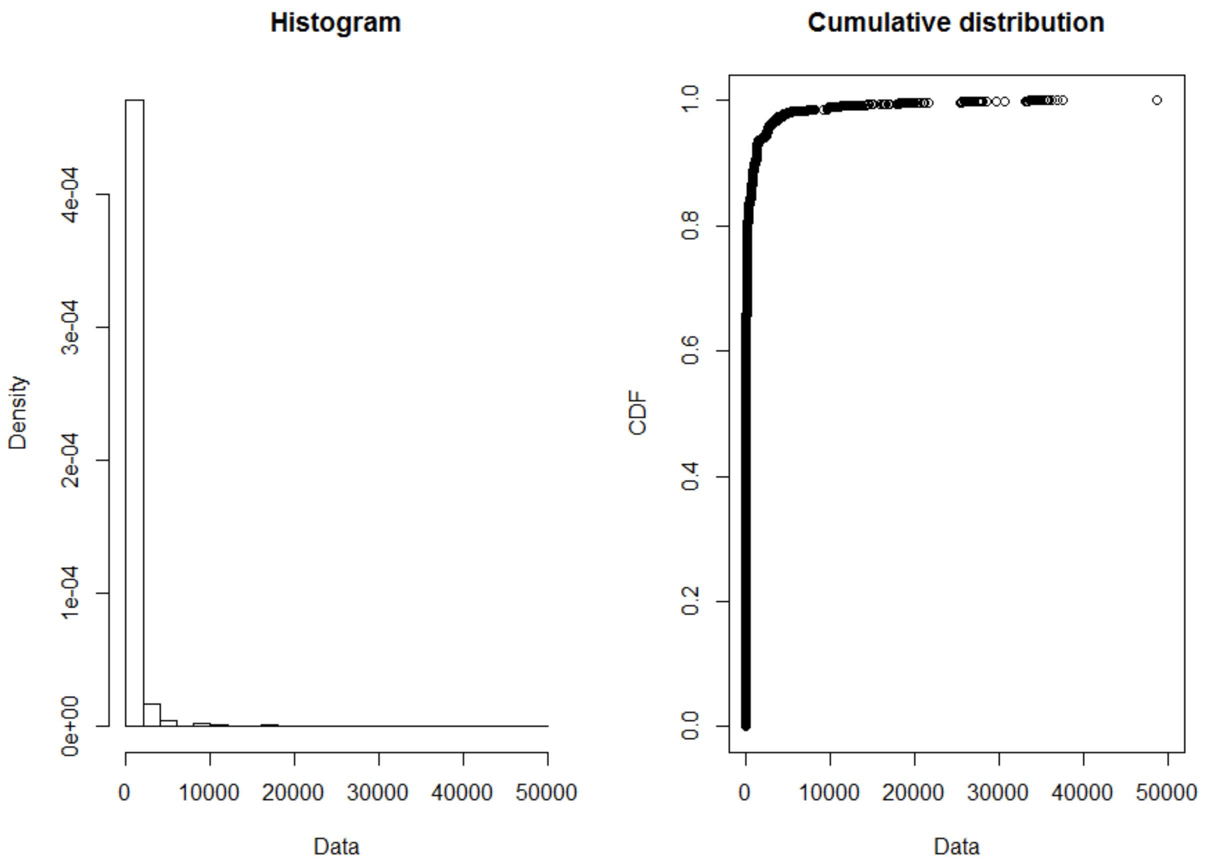
As you can see from Figure N°9, it is possible that the industrial company does not record any loss realization because we mainly study rare risks. However, if the risks manifest themselves we have to deal with colossal amounts of the order of a few hundred million MAD.

**Figure N°10 : Histogram and cumulative probability distribution**




**Source : Established using R software**

From Figure N°10 we can see that more than 90% of the loss amount scenarios are less than 10,000 Million MAD. This value is very far from the average loss calculated from the database.

## Conclusion

At the end of this work, the qualitative study of the industrial risks of the industrial company carried out on the database provided by the expert allowed us to conclude that we are facing major risks.

We have given a detailed and original in-depth study of the empirical modeling of annual industrial losses for the case of Moroccan industrial companies. Indeed, we thought of deploying a whole procedure of modeling these risks by sweeping all the possible methods and by explaining the limits of the approaches which did not allow us to achieve satisfactory results.

At the end of this step we released two models, namely the normal approximation of the loss distribution and the Monte Carlo simulation through calculation algorithms coded on the R software. We chose the model resulting from the Monte Carlo simulation combined with the flat-rate approach as the model most faithful to the loss distribution by scanning all possible loss scenarios. We have adopted this approach because it allows us to glimpse the different loss scenarios which will present a good basis for decision-making for the company's risk management teams.

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